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Structuring and Adjusting Content for Students: A Study of Live and Simulated Tutoring of Addition

Ralph T. Putnam
Michigan State University

This study examined how six experienced teachers acquired information about students' knowledge and used that information to adjust their instruction while tutoring. Each teacher tutored five simulated students and one live student in the algorithm for whole number addition. A diagnostic/remedial *perspective in which the teacher forms a detailed model of the individual student's knowledge and misconceptions was assumed in the early stages of the study, but did not describe adequately the tutoring of the teachers. Diagnosis was not their primary goal. Rather, each teacher appeared to move through a* curriculum script*—a loosely ordered but well defined set of skills and concepts students were expected to learn, along with the activities and strategies for teaching this material.*

Teachers are often urged to adjust their instruction to the needs and skills of individual students. To make such adjustments, teachers must infer in some way what students know. Although such inferences may take place through more or less formal methods, such as examination of tests or written assignments, they also form an important part of interactions with students during ongoing instruction. How does a teacher decide, for example, when a student has learned a particular concept or procedure and is ready to go on? When a student makes an error, does the teacher assume the error was a careless one and ignore it? Or does the teacher stop to explain some concept or procedure, believing that the error reflects some underlying misconception?

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The importance of this aspect of teaching has been pointed out before. Rosenshine (1983) included "check for understanding" as one of his basic teaching functions. Good and Grouws (1979) similarly included "assess student comprehension" as one of their instructional behaviors for effective mathematics teaching. Shavelson (1978) argued that it is important for teachers to estimate the "states of mind" of their students and that "these estimates provide essential information for deciding *what* and *how* to teach" (p. 40). Brown and Burton (1978) argued that:

> One of the greatest talents of teachers is their ability to synthesize an accurate "picture," or model, of a student's misconceptions from the meager evidence inherent in his errors. *A detailed model of a student's knowledge, including his misconceptions, is a prerequisite to successful remediation* [italics added]. (pp. 155–156)

Developing computer programs that can form such detailed models of students' knowledge has been one of the major challenges in developing "intelligent" computer-assisted instruction (Sleeman & Brown, 1982). It is hoped that such models of student knowledge will permit more flexible and relevant instruction.

Mathematics educators have similarly spoken about *diagnosis* during instruction. Buswell (1926) argued for understanding how students are thinking to avoid "blind teaching of the whole class without knowledge of the specific needs of the individual pupils" (p. 86). In more recent advice to mathematics teachers, Ashlock (1982) urged that "diagnosis should be characteristic of your instruction in computational procedures. In fact it should be typical of *all* of your instruction. You need to be alert to what each child is actually doing and eager to probe deeper" (p. 9).

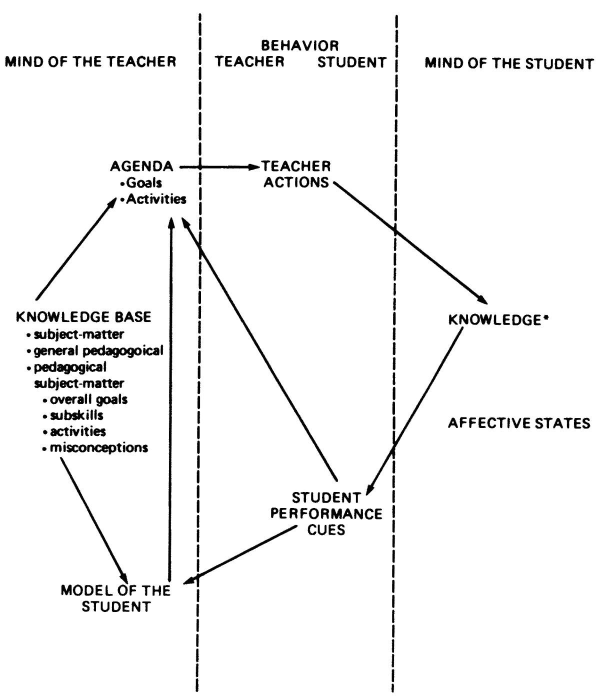
In spite of the attention that diagnosis has received, how teachers determine what students know and how they use this knowledge to adjust the content of instruction have not been systematically explored. Most work in this area either has been prescriptive—suggesting what teachers *should* do—or has *assumed* that teachers make the necessary inferences and adjustments. We know little about how teachers actually determine what students know during interactive instruction. The nature and roles of teachers' thought processes and knowledge of the subject matter in adjusting instruction to the knowledge of individual students are similarly unclear.

The current study began as an investigation of teachers' diagnostic strategies as they tutored individual students in the skills of whole number addition, an important component of the elementary mathematics curriculum. Each teacher tutored a second-grade student and several "students" simulated by a computer. The original purpose was to observe experienced teachers in tutorial interactions to see what kinds of diagnostic strategies they used and how they adjusted their instruction on the basis of their diagnoses. Underlying this purpose was the assumption that diagnosis

would be an important part of the teachers' interactions with individual students. The diagnostic strategies that teachers used might be less than optimal, but the goals of determining the nature of a student's difficulty and then correcting the source of that difficulty were assumed. As the study progressed, however, it became clear that detailed diagnosis was *not* an important goal of the teachers as they tutored students. Diagnosis of a student's difficulties and remedial teaching to correct those difficulties were not clearly separable. The result was a shift in the study's focus from describing specific diagnostic strategies to examining how the teachers structured and sequenced subject-matter content for their students.

A Model of Tutoring

The general model of tutoring that guided this research is presented in Figure 1. The model draws on previous models of teacher thinking and decisionmaking (Leinhardt & Greeno, 1986; Shavelson & Stern, 1981) and on Collins and Stevens's (1982) analysis of inquiry teaching. The model covers three domains: (a) the mind of the teacher, (b) the mind of the



*The student's knowledge includes prior conceptions, schemata, etc.

FIGURE 1. *General model of tutoring*

student, and (c) behavior of the teacher and the student. Because the emphasis of this study is on how the teacher functions, the student side of the model is sparse, consisting primarily of *knowledge. Affective states* of the student are included because they undoubtedly have an influence; they were not, however, addressed in this research.

The key elements within the mind of the teacher are an *agenda*, a *knowledge base*, and a *model of the student*. The agenda is the teacher's plan for the lesson: a dynamic plan that changes during the course of the lesson or tutorial session as the teacher obtains new information about the student and draws upon previous knowledge. More than a teacher's written plan for a lesson, the agenda includes a teacher's goals, subgoals, and activities for a lesson (Leinhardt & Greeno, 1986). It is the goals and subgoals of the agenda that determine the *teacher's actions*. The agenda, in turn, is shaped by the teacher's knowledge base, the model of the student, and input from the student in the form of student performance cues (from which the teacher must infer what the student does or does not know).

Several components of the teacher's knowledge base are thought to play important roles in tutoring (cf. Shulman, 1986). *Subject-matter knowledge* is knowledge of the domain being taught—in this study, knowing how to add. This knowledge is not unique to teachers. *General pedagogical knowledge* consists of facts and theories about how students learn and general ways of presenting information and structuring lessons. This knowledge is not limited to specific content areas. *Subject-matter-specific pedagogical knowledge* concerns the teaching of a particular domain (e.g., addition of whole numbers). The experienced elementary teacher has a store of activities, examples, memories of previous students, and so forth, all dealing specifically with the teaching of addition. This knowledge is unique to teachers; it is not held by experts in mathematics or by lay persons. Considering in more detail the nature of this subject-matter-specific pedagogical knowledge was an important part of the current study.

It is assumed that the teacher must have some sort of model of the student—the individual student being tutored. This model influences the teacher in deciding whether to move on to new material, give more practice in a certain kind of problem, or probe a student for understanding. Whereas the teacher's model of the student includes information about both cognitive and affective aspects of the student, only the cognitive aspects (i.e., the student's knowledge) are considered in this study. Brown and Burton (1978) have suggested that the teacher's model of the student's knowledge is (or should be) a detailed representation of his or her misconceptions or faulty procedures.

Specifics about the processes occurring during tutoring can be postulated within the framework provided by the general model in Figure 1. The orientation or approach to teaching that guided the planning of this study

and that is implicit in much writing and research on diagnosis (e.g., Ashlock, 1982; Brown & Burton, 1978) will be referred to here as the *diagnostic/remedial* approach. In this approach the teacher's model of the student, which is modified by input from student performance cues, is largely responsible for driving the teacher's agenda for a lesson. In other words, what the student knows (as perceived by the teacher) is the main determinant of topics and their sequence in the tutoring lesson. The first main goal of the teacher in this approach is to determine the underlying cause of a student's errors—in the form of misconceptions or faulty procedural knowledge. Once the source of errors is determined, the goal becomes correcting the underlying faulty knowledge through instruction. It has been argued that this kind of diagnostic/remedial approach, by taking the individual student's knowledge as the starting point, provides optimal learning opportunities for the student.

The diagnostic/remedial approach did not, however, provide an adequate description of the tutoring of the teachers in this study. Rather than being driven primarily by the teacher's model of the student, the content of a tutoring session appeared to be driven much more by the teacher's knowledge of and goals for teaching addition skills. As a result, an alternative to the diagnostic/remedial approach was developed. From this alternative perspective, the teacher has a *curriculum script*—a loosely ordered but well-defined set of skills and concepts students are expected to learn, along with the activities and strategies for teaching this material. The teacher's primary goal, rather than diagnosis per se, is to move through this organized and sequenced set of skills and activities, gathering enough information from student performance cues to correct any student difficulties or misconceptions that might arise. Thus, although student input is important, the curriculum script is the major determinant of the agenda for the tutoring lesson. A model based on the curriculum script construct will be discussed in detail after the procedures and results of the study from which it was developed have been described.

Method

Subjects

Four second-grade and two first-grade teachers from public schools in the San Francisco Bay Area participated in the study. All were women with at least 10 years of teaching experience at the elementary level. Teachers were recruited to include as wide a range of backgrounds and approaches in the teaching of mathematics as possible. Some were recommended by their principals as being strong mathematics teachers who had been involved in various inservice and curriculum development activities. Others agreed to participate in the study because they were interested but did not consider themselves to be particularly outstanding mathematics

teachers. The teachers received monetary compensation for their participation.

Live Tutoring

Each of the experienced teachers tutored one second-grade student. The second-grade teachers (Alma,[1] Betty, Carol, and Donna) tutored students from their own classrooms; the first-grade teachers (Ella and Fran) tutored students they had taught in the previous year.

Selection of students. The students to be tutored were selected on the basis of their performance on an addition test administered to each teacher's class. The test consisted of 12 addition problems designed to reveal common student difficulties. The tests were scored and several students making systematic computational errors were identified as possible tutoring candidates. The teacher made arrangements to tutor one of these students.

The live tutoring task. Each teacher tutored one student during two 20-minute sessions before or after school. The task for the tutoring sessions was explained to each teacher through a set of written instructions and a briefing session lasting approximately 30 minutes. The goal stated in the written instructions was "to teach the student to add two numbers, each having up to three digits, with renaming (carrying)." Examples of the sorts of problems the student was expected to work were given, along with the qualification that the teacher should adjust the level of difficulty as appropriate for the student. The teacher was told to use any procedures or techniques thought to be appropriate and to bring to the tutoring session any written or manipulative materials that might be needed.

The tutoring sessions were conducted in the teacher's classroom, with the teacher and student seated side by side at a table. A video camera was focused on the written work of the teacher and student. A timer was set to notify the teacher when the 20 minutes allocated for the session had transpired. I was present during the tutoring sessions to monitor the video equipment and deal with any interruptions or questions that might arise.

Stimulated recall. After each tutoring session, I viewed the videotape with the teacher, stopping the tape to ask questions and discuss various aspects of the tutoring. Each stimulated recall session, which was tape-recorded, took place within a few minutes of the tutoring. The stimulated recall interviews were loosely structured. Their purpose was to gain insight into the teacher's goals, strategies, and possible hypotheses about the student by allowing the teacher to discuss and clarify her goals and actions. Although some researchers have used stimulated recall as a primary source of data on teachers' thoughts during interactive teaching (e.g., Marland, 1977; Peterson & Clark, 1978), the technique was used here only to

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[1] All teacher names are pseudonyms.

complement data from the tutoring sessions themselves, not to create a complete protocol of the teacher's thoughts while tutoring. The tape was stopped when the teacher remembered something significant about what she was thinking during the tutoring session and at points where the teacher had given the student problems to work or completed some set of actions, such as correcting an error or giving an explanation. The following questions were typical of those asked: "Is there anything significant you can remember here?", "What were you trying to do here?", "Was there any particular reason for giving that example?", and "What did you think the problem was here?" (after a student error).

Simulated Tutoring

After completing the two live tutoring sessions, each teacher tutored six computer-simulated students in a session lasting approximately 2½ hours. The following description of the simulation is brief; more detail may be found in Putnam (1985).

The simulation. For the simulation,[2] the computer is programmed to represent a series of individual students who make various systematic errors working addition problems posed by a live teacher. Each simulated student uses an incorrect, or *buggy*, procedure for carrying out the steps of the addition algorithm. Descriptions of the buggy procedures and the resulting errors of each simulated student may be found in Table 1. To insure that the errors were like those made by actual students, only errors that have been reported in at least two empirical studies (Cox, 1974; Graeber & Wallace, 1977; Roberts, 1968) were included.

The teacher interacts with each simulated student by generating and presenting addition problems and selecting from a predetermined set of instructional *moves.* The moves are various statements and activities a teacher might use when teaching addition. Each move is written on a card with an identifying number, which the teacher types into the computer (see Table 2). The simulated student responds correctly or incorrectly to the addition problems posed by the teacher (the student's written work appearing on the screen) and "learns" from the instructional moves that address the student's buggy procedures.

The instructional moves and their effects on simulated students are based on recommendations for remedial teaching by various authors (Ashlock, 1982; Reisman, 1978; Resnick, 1982). Each instructional move addresses one or more subprocedures of the addition algorithm (e.g., carrying) and results in the learning of those procedures by the simulated student. A move is either *algorithmic* or *conceptual.* An algorithmic move describes or demonstrates the procedures of addition without explaining

[2] The simulation was patterned largely after the BUGGY system of Brown and Burton (1978) and a teaching game by Moore (1973).

TABLE 1
Errors made by simulated students

Student	Bug	Description of buggy procedure	Examples		
I	(none)	—	1 56 +9 65	1 33 +29 62	12 +23 35
II	Omit-Carry	Does not write carried digit at top of next column.	56 +9 55	33 +29 52	12 +23 35
III	Cross-Add	Adds digits in a "short" column by replacing "missing" digits with digits from next column.	53 +5 108	24 +3 57	1 24 +26 50
IV	Matched student[a] (Write-Both-Digits)	Writes[a] both digits of a sum greater than 9 in answer instead of carrying.	56 +9 515	33 +29 512	12 +23 35
V	Random-Basic-Fact	Randomly generates sums one larger or smaller than correct basic fact. These errors are made only when both digits are greater than 5 and not "doubles" (e.g., 6 + 6).	1 76 +18 95	1 37 +19 56	1 16 +19 36
VI	Add-Digits-Separately & Omit-Carry[b]	Adds all digits without respect to position.	23 +16 12	15 +6 12	3 +2 5

[a] The errors made by Student IV were matched when possible to the errors made by the live student tutored by the teacher. Write-both-digits was used to replace other bugs if needed for the matched student.
[b] Student VI's Omit-Carry errors occurred only after the add-digits-separately bug was corrected.

the reasons for the various steps or linking the procedures to the conceptual knowledge of the student. The teaching remains at a mechanical or rote level. These algorithmic moves result in temporary learning that may be subsequently forgotten by the simulated student. A conceptual move is intended to help the student understand the addition process. It links the procedures involved to mathematical concepts by providing reasons for what is done or by using concrete materials. Conceptual moves result in

TABLE 2
Teaching moves in simulated tutoring

Move	Description	Example
Example	Student is to work the addition example the teacher enters (up to three one- to three-digit addends).	*23 + 46 + 19*[a]
Algorithmic instructional		
Demonstrate	Teacher works through the specified problem, talking through the steps taken. No explanation of the steps is offered.	*DEMO 89 + 16*
Describe	Teacher describes the steps in a procedure (possibly demonstrating the steps). These moves are written out for the teacher, who types only the move number.	*M7* "When a column adds up to more than ten, you have to write the ones and carry the tens like this (demonstrate writing and carrying the proper digits)."
Conceptual instructional		
Concrete	Teacher helps the student work problems with concrete objects (e.g., sticks and bundles), relating the steps taken to the steps in the written algorithm. These moves are written out for the teacher.	*M11* Have the student use sticks/beans and bundles/cups of 10 to add numbers greater than 10 that *require* regrouping (e.g., 34 + 28). Make new bundles/cups to represent regrouping. Show how the written procedure of carrying corresponds to this step. Have the student practice adding with the sticks/beans and the written procedure until he or she can do them correctly.
Explain	Teacher explains the steps in a procedure, giving the reasons for what is done. These moves are written out for the teacher.	*M20* "These are all ones (point to the ones column) so you add them together and write the answer. These are all tens (point to the tens column) so you add them together. You can't add the tens and the ones together."

TABLE 2 (continued)

Move	Description	Example
Dissonance	Teacher has the student work a problem two different ways and compare the answers to create dissonance. These moves are written out for the teacher.	*M10* Have student add 15 + 18 on paper, using the method used in the last example he or she worked. Then have student add 15 + 18 by counting individual sticks/beans. After having student compare the two answers, have student work the written problem again, emphasizing the importance of adding the ones column first, then moving to the tens to get the answer obtained by counting.
Other		
Nextday	Teaching for the day is concluded and resumed on the following (simulated) day.	*NEXTDAY*
Quit	The teaching session is terminated. This move is used when the teacher is satisfied that the current student has mastered the target skills.	*QUIT*
Review	Allows tutor to see previous moves and student's work. This move has no effect on the student.	*REVIEW*

[a] Italic indicates word or move number that teacher types on computer keyboard.

permanent learning of the skills they address (i.e., once learned, the correct procedure cannot be forgotten by the simulated student). Examples of algorithmic and conceptual moves are given in Table 2. Note that whereas the instructional moves and their effects are based on simplified assumptions about how students learn, they present a range of realistic instructional options to the teachers who were the focus of this study.

The teacher works with each simulated student by entering addition problems and instructional moves until satisfied that the student has mastered the addition algorithm. At that point, the teacher is presented with a retroactive *prediction task*. The computer presents four addition problems and the teacher is asked to work the problems as the simulated

student would have worked them at the *beginning* of the tutoring session, before any of the student's difficulties were corrected. This task is used to assess the accuracy of the teacher's diagnoses about the kinds of errors the simulated student made.

Procedure for the simulated tutoring. Each teacher read a set of written instructions describing the simulated tutoring task and how the various moves worked. The teacher spent a few minutes becoming familiar with the various teaching moves, arranging the cards on the table to facilitate locating them during the tutoring. The teacher worked at the computer with a practice simulated student who made no errors, trying out the various moves and becoming familiar with how the simulation worked. The teacher then tutored the remaining five simulated students, predicting responses for each student after tutoring. Three questions were asked of the tutor after working with each simulated student:

1. What was _______'s problem?
2. When you decided what the problem was, how did you try to teach her (him)?
3. If you had been working face to face with _______, how would your approach have been different?

Written notes were made on the tutor's responses, and the discussion was tape-recorded. Following the tutoring of Student IV, the interaction with that student was replayed by the computer. Each move made by the teacher and the student's response appeared on the computer screen. The teacher was asked questions similar to those in the stimulated recall for the live tutoring to clarify goals and strategies.

Analysis

A complete transcript of each live tutoring session was made to include all teacher and student talk, detailed descriptions of teacher and student actions, and comments made during stimulated recall. The computer produced complete records of the simulated tutoring sessions. Each record consisted of the teaching moves selected by the teacher (including the specific addition problems the teacher generated) and each simulated student's responses and knowledge state before and after each teaching move. Audiotapes and notes about the teacher's responses to various questions during the simulated tutoring and stimulated recall were available to supplement the record of the simulated tutoring.

Following transcription came a period of cycling among data reduction, data display, and conclusion-drawing/verification, three streams of activity in qualitative data analysis posited by Miles and Huberman (1984). As in protocol analysis methods used in psychological studies of problem solving (Ericsson & Simon, 1984; Newell & Simon, 1972), the intent was to extract patterns of goals and actions from sequences of behavior (tutoring in this case).

Three aspects of the interaction in the live tutoring were selected to focus the inquiry: (a) how the teachers responded to student errors and deficient responses, (b) how teachers elicited student responses that provided student performance cues and structured the content of the tutoring sessions, and (c) the teachers' overall goals for the tutoring sessions. Data from the simulated tutoring were used to support and confirm hypotheses developed from analysis of the live tutoring.

Results

Response to Errors

Student errors and deficient responses (incomplete responses or failures to respond) provided an ideal site for studying how the teachers reacted to possible student misconceptions and made adjustments in their instruction. Errors and deficient responses carried potential information about erroneous or incomplete knowledge of students. They were among the most salient cues for the teachers in the live tutoring because they marked a departure from the generally expected correct performance (i.e., teachers virtually always reacted in some way when students were not performing correctly).

Table 3 displays the frequency of errors and deficient responses made by each student during the live tutoring. These errors and deficient responses occurred while students were working addition problems or carrying out parts of the addition procedure in response to prompting questions by the teachers (e.g., "What is 5 + 3?" or "Would you write your answer down?"). Whether the errors of an individual student represented systematic faulty procedures or misconceptions cannot be determined because errors were in most cases corrected by the teacher as they occurred.

The teachers' responses to the errors and deficient responses were examined and classified in terms of subgoals and strategies. *Strategy* here refers to a pattern of actions to reach a subgoal. The teacher may or may not have been conscious of using a particular strategy; the patterns were inferred from the teachers' actions. The subgoals and strategies of the teachers are described in Table 4. Table 5 presents the frequency of occurrence of the subgoals and strategies for each teacher.

The diagnostic/remedial approach described earlier suggests that when a student makes an error, the teacher should understand the nature of the error and what is causing it before trying to correct it. Brown and Burton (1978), for example, warned against the detrimental effects on students of teachers attributing errors to the wrong causes, pointing out that incorrect diagnoses often result from jumping to a conclusion before collecting enough data. Following only eight (7%) of the 107 errors and deficient responses in the live tutoring, however, did teachers appear to hold the distinct subgoal of determining the nature of a student's difficulty. Fran,

TABLE 3
Frequency of errors and deficient responses during live tutoring

Error types	Specific errors	Student						Total
		A	B	C	D	E	F	
Working Problems								
Deficient response	Incomplete response; failure to respond	4	—	—	6	—	1	11
Summing	Basic-fact error; omits digit when summing	1	4	2	2	4	5	18
Carrying	Omit-carry[a]; write-both-digits[a]; carries when not needed	1	—	2	—	11	1	15
Place-Value	Misaligns columns (including left justification of addends); omits zero when writing number; reverses addends to avoid having smaller addend on top; answer digits not in proper columns	8	2	—	—	6	2	18
Column-Sequence	Works left-to-right or other incorrect sequence[b]	7	—	—	—	1	—	8
Wrong operation	Subtracts instead of adding	—	—	2	—	—	—	2
Minor clerical	Omits minus sign, comma, or end-of-problem line; miscopies problem	3	3	2	—	1	1	10
Prompting Questions								
Oral basic fact questions								
Deficient response	Fails to give answer	—	—	—	1	—	—	1
Error	Incorrect sum, e.g., 6 + 7 = 12	—	—	1	8	1	—	10
Read number request								
Deficient response	Fails to read number	1	—	2	—	—	—	3
Error	E.g., "four hundred" instead of "four thousand"	3	3	—	—	1	—	7
Concrete prompt								
Error	Error in response to request to perform specific manipulative task, e.g., miscounting beads.	—	—	—	1	—	3	4
TOTAL		28	12	11	18	25	13	107

[a] Error was made by simulated students and is described in Table 1.
[b] Working from left to right was considered an error by teachers whenever it occurred, even though it does not always result in an incorrect answer.

TABLE 4

Subgoals and strategies in teachers' responses to errors and deficient responses during live tutoring

Subgoal	General strategy	Particular strategies
Get the correct answer to the problem being worked	Have the student correct the error and move on	Give correct answer Indicate necessary change Signal error (indicate that there is an error and possibly where it is without saying what the error is or how to correct it Talk through problem until student sees error Ask Algorithm Action or Student Process question to signal error
Teach fact, procedure, concept, etc.	Follow correction of error (using one of above strategies) with some other treatment of the process or concept involved	Ask questions to be sure student knows procedure or concept Explain procedure or rule Demonstrate working problem incorrectly to show that wrong answer will be obtained
	Teach fact, procedure, or concept while getting the particular error corrected	Give explanation or point out rule to indicate error Model correct procedure while pointing out error Ask questions to lead student to realize that error has been made Have student compare with another problem or procedure to realize that error has been made (For deficient response) ask prompting questions to provide model and support for student in working problem
	Accept answer; teach fact, procedure, or concept later	
Determine nature or extent of student's difficulty	Ask student process questions Allow student to work problem incorrectly to see if student realizes error has been made	

TABLE 5

Percentages (frequency) of subgoals and strategies in teachers' responses to errors and deficient responses during live tutoring

Subgoals and strategies	Teacher						Total
	A	B	C	D	E	F	
Subgoal: Get correct answer to current problem (without addressing more general fact, procedure, or concept)	29(8)	75(9)	45(5)	61(11)	88(22)	54(7)	58(62)
Have the student correct the error and move on	29(8)	75(9)	45(5)	61(11)	88(22)	54(7)	58(62)
Subgoal: Teach fact, procedure, concept, etc.	68(19)	25(3)	54(6)	39(7)	12(3)	46(6)	41(44)
Follow correction with other treatment		25(3)	18(2)	6(1)		8(1)	7(7)
Teach while correcting error	57(16)		36(4)	33(6)	12(3)	38(5)	22(34)
Accept answer; teach later	11(3)						3(3)
Subgoal: Determine nature/extent of error	11(3)		27(3)			15(2)	7(8)
Ask questions	4(1)		27(3)[a]			15(2)[a]	6(6)
Allow student to work incorrectly	7(2)[a]						2(2)
Total number of errors	(28)	(12)	(11)	(18)	(25)	(13)	

Note. Entries in the Subgoal rows represent sums of entries in rows of subsumed strategies. More than one subgoal and strategy may apply for a single error, resulting in percentages for a single teacher totaling more than 100.

[a] Teacher also had subgoal of correcting answer or source of error.

Carol, and Alma were the only teachers who at times clearly held this subgoal, using the strategy of asking students what they had done or what they were thinking about [e.g., "What were you thinking when you added that?" (Fran); "Now, did you add the ones column?" (Carol)]. Alma also used the strategy of allowing her student to work problems incorrectly to see if she would catch her own errors.

Rather than determining the exact nature of students' errors, the primary subgoal of the teachers when students made errors or failed to work problems was to get the correct answer to the particular problem being worked. After virtually all the errors made by students, the teacher worked to be sure the student at some point got the correct answer. Following 58% of the errors (see Table 5), getting the correct answer to the particular problem being worked appeared to be the *only* subgoal of the teacher; there was no apparent attempt to teach a fact, procedure, or concept more general than the particular problem.

When a teacher's only subgoal was for the student to get the correct answer, the strategy generally employed was to get the error corrected and

move on to another problem. The teacher might simply give the correct answer or indicate the necessary change [e.g., "Start from here" (Ella)], or signal the existence of an error [e.g., "There's one mistake in here somewhere in your adding" (Betty)]. At times, teachers continued to work at getting correct answers for particular problems without attempting to correct the source of the errors even when students were having considerable difficulty. For example, when Ella's student had difficulty writing digits in the proper columns (place-value errors), Ella repeatedly told the student where to put the misplaced digits. She did not discuss place-value concepts, even when the student made four different errors writing a single problem. Donna responded to all eight of her student's oral basic-fact errors by signaling the error with "close," or a hint like, "No, 20's too much." The student proceeded to try again with another answer, the interchange taking on the character of a guessing game until the correct answer was produced.

Following 41% of student errors (see Table 5), teachers had in addition to the subgoal of getting the correct answer the subgoal of teaching a fact, procedure, or concept more general than the particular problem being worked. A teacher with this subgoal tried somehow to correct the student's knowledge. The knowledge of focus might be a basic fact, part of the addition procedure, or a place-value concept.

One could argue that when a teacher addressed something more general than an error in a particular problem, a diagnosis was implicit; some hypothesis about what was causing the error must have preceded the attempt to correct its source. What is significant here is that the teacher's goal appeared to be teaching the correct fact, procedure, or concept—not determining more precisely the nature or source of the student's error. The teaching of correct content generally was not preceded or accompanied by teacher actions that implied the subgoal of determining more precisely the nature or extent of the student's difficulty. Thus, either the source of the student's error was obvious to the teacher without further diagnosis or the teacher's response to the error was not dependent on knowing precisely the source of the error.

In working with the subgoal of teaching the correct fact, procedure, or concept, teachers either followed correction of the error with some further treatment of the topic or addressed it in the process of getting the error corrected. In either case, the amount of emphasis placed on teaching the correct information might be minute or considerable. A teacher might make a single statement to remind the student of a rule, or she might spend several minutes discussing some aspect of the problem.

The teacher's emphasis in teaching the correct information might be at an algorithmic level or at a more conceptual level. Such differences in approach are illustrated by Betty's and Alma's treatment of place-value errors like those to which Ella responded by simply telling the student

where to put the digits. When Betty's student lined his addends up on the left in writing a problem, Betty first directed him to correct his digits, telling him where to put them when he had difficulty. After the student finished working the problem, Betty demonstrated what would happen if the problem were added while left-justified, actually working the problem incorrectly. She gave the algorithmic-level explanation that not keeping the columns straight would result in the wrong answer, but made no mention of place-value concepts. When Alma's student made left-justify errors, Alma put forth considerable effort, sometimes spending several minutes getting the student to realize that place-value constraints had been violated without directly telling the student what error had been made. In two cases, Alma asked the student about the place values of the digits in the addends until the student realized that she had put ones and tens in the same column. At other times, Alma let the student work a left-justified problem incorrectly and then had the student estimate the answer or compare it to another problem. Thus, Ella, Betty, and Alma responded to similar place-value errors in different ways. Ella focused solely on getting the individual problems corrected. Donna held this subgoal, but also worked to correct the student's knowledge at an algorithmic level. Alma used a much more conceptual approach.

In sum, during the live tutoring, the teachers did not respond to student errors by probing for the exact nature of the students' difficulties as would be suggested by the diagnostic/remedial approach. Rather, a teacher worked for the student to get the correct answer to the problem being worked and, possibly, to teach the correct procedure, rule, or concept. Once the error was corrected and the student's performance considered satisfactory, the teacher moved on to another problem.

In the simulated tutoring as well, a teacher's usual subgoal was to get errors corrected and move on to other problems, *not* to determine in detail the nature or extent of the student's difficulty. In the simulated tutoring, a teacher with the subgoal of further diagnosing a student's difficulty could give additional problems for the student to work before trying to correct the source of the error by teaching the correct procedure with an instructional move. Indeed, the simulation was designed with the implicit assumption that when confronted with a student error, a teacher would give additional problems until the nature of the student's buggy procedure was clear. The teacher would follow this diagnosis with various instructional moves to correct the bug. After only two of the many errors made by simulated students, however, did a teacher follow an error with another problem for the student to work. In all other cases the teacher immediately followed errors with instructional moves, attempting to remedy the student's difficulty. (Whether an instructional move resulted in temporary or permanent learning by the simulated student was determined by whether the move was *algorithmic* or *conceptual* and by whether it addressed the

faulty addition subskill.) After presenting an instructional move, the teacher generated similar problems to see whether the student's difficulty had been corrected. If it had not, the teacher tried different instructional moves until the student stopped making errors. Never did a teacher generate further problems to pinpoint a student's difficulty after an instructional move failed to remedy the student's difficulty.

Eliciting Student Responses

Although detailed diagnosis following student errors was not a primary goal for the teachers, gathering information about what a student knew and could or could not do was an important part of the teachers' activity during the live tutoring. Because the live tutoring sessions were one-on-one situations much like conversations, virtually every move or action by a teacher was followed by some sort of student response providing cues about the student's knowledge. The general pattern was one that has been well documented in studies of verbal interactions in classrooms: *initiation* by the teacher, *reply* by the student, and *evaluation* by the teacher (Mehan, 1979; cf. Bellack, Kliebard, Hyman, & Smith, 1966). Although almost all moves of the teacher involved acquiring cues from students, this goal was not clearly separable from other goals of providing practice or getting correct information stated. For example, a teacher might have the student work an addition problem requiring carrying, fulfilling both the goal of providing practice and the goal of checking on the student's ability to do the carrying procedure. Or the teacher might ask a series of questions to get the student to state rules for the addition algorithm, simultaneously checking the student's knowledge of the rules and getting the rules stated correctly. This feature of the live tutoring was inconsistent with the diagnostic/remedial approach, which separates the activities of diagnosis and reteaching.

Two distinctive ways of eliciting responses from students emerged during the tutoring sessions: (a) having the student work various addition problems and (b) asking questions or otherwise requesting a response from the student. It was through their requests for student responses that the teachers structured the lessons and moved from one topic to the next.

Working problems. Because a large part of addition knowledge is procedural in nature, teachers gave students problems to work. Working these addition problems was the central activity of the tutoring sessions. Teachers varied the problems on several *problem features*, shown in Table 6. In giving problems with particular features, a teacher simultaneously checked on the students' ability to carry out the addition algorithm with that class of problems and provided carefully monitored practice of the procedures involved. Although the teachers differed in how systematic they were in varying problem features, they generally began with simple problems, gradually adding more digits, addends, and carrying. The typical progres-

TABLE 6

Problem and task features varied by teachers in live tutoring

Feature	Subgoal	Teacher[a]					
		A	B	C	D	E	F
Problem feature							
Number of addends	—	/	/	/	/	/	/
Number of digits	—	/	/	/	/	/	/
Carrying	—	/	/	/	/	/	/
Passing: carrying from leftmost column (passing from ones column not included)	—	/	/	/	/	/	
Nonrectangular problem: different number of digits in addends	Student keeps digits in proper columns when writing problems	X	X	X		/	/
Carrying digit larger than 1	Make sure student does not always carry 1		/		X	/	
Zeros in addend	Student can add columns containing zero	X	X	/		/	X
	Student can write number containing zero correctly				X	X	
Individual digits varied for certain combinations	Student uses correct strategy for combining digits					X	X
	Help student see "secret of 9," etc.						X
Task feature							
Basic format: teacher writes problem in vertical format and student works	Student can perform steps of algorithm	/	/	/	/	/	/
Student copies problem from vertical format to vertical format or from horizontal format to vertical format	Student can copy and keep columns aligned				/		
Nonrectangular problems	Student keeps digits in proper columns when writing problem	X		X			

TABLE 6 (Continued)

Feature	Subgoal	Teacher[a]					
		A	B	C	D	E	F
Rectangular problems	Student knows to rewrite horizontal problems to vertical before adding			X	X		
Teacher dictates problem and student writes it in vertical format							
Nonrectangular problems	Student keeps digits in proper columns when writing problems	X				/	
Rectangular problems	Student can write correctly numbers containing zero					X	X
in horizontal format	—[b]				/		
Student works problem in horizontal format							
Using nonstandard written algorithm	Teach new procedure[a,b]				X		
Working mentally	Student can do "mental math"		X				
Student makes up own problem given certain constraints	Student has to think ahead and understand carrying		X				

[a] Teachers are identified by initials, e.g., A = Alma; X = teacher made comment or action clearly indicating subgoal; / = feature used, but subgoal not clear.

[b] Donna taught student a new procedure for adding in horizontal format during the second tutoring session. This part of the session was not fully considered in the analysis.

[c] When Carol gave problems in horizontal format, she expected the student to copy them into vertical format. The student, however, did work some problems horizontally without copying.

sion for carrying was to move from no carrying to carrying from the ones column, to carrying from both the tens and ones columns. Other problem features that were varied by some but not all of the teachers are listed in Table 6, along with the specific subgoals teachers had in mind when introducing these features.

When teachers varied problem features, they sometimes commented in stimulated recall interviews that they thought the student might have difficulty, indicating their knowledge of likely student difficulties or misconceptions. Donna, for example, said, "Any like *this*, where there's a zero

in the tens [may cause difficulties in writing the number]." Commenting on giving her student a problem with zeros in the addends, Alma said, "I wanted to see if she could carry across the zero [i.e., would write the carried digit at the top of a column containing a zero]." Thus there was an awareness of some of the things that might be difficult for the student. Note, however, that the teachers talked about the anticipated difficulties in terms of the *correct* procedure (i.e., whether the student would or would not be able to do something correctly) rather than in terms of the kind of incorrect procedure the student might use.

Teachers also varied *task features*—how the problems were presented and what the students were required to do to work them. The basic format used by all of the teachers was for the teacher to write the problem on the student's paper for the student to work. But teachers sometimes presented problems in different ways to vary the student's task. For example, a teacher might present the problem orally or in a written horizontal format and have the student write it in the standard vertical format before adding, thus making the student's task more complex. The task features individual teachers used and their accompanying subgoals are presented in Table 6.

Thus, in the live tutoring, the teachers moved through systematic sequences of problem types, varying problem and task features as they moved from simple to more difficult problems. In the simulated tutoring, the teachers structured the problems they presented to the simulated students in a similar fashion. Each teacher tutored five simulated students, revealing consistencies in each teacher's approach in teaching students making different kinds of errors. As in the live tutoring, the pattern was to move from simple to more difficult problems by adding new problem features, usually one at a time.

As an example of this consistency, Figure 2 displays the sequences of problem types generated by Donna during the live and simulated tutoring. Descriptions of the addition problem types are listed in Table 7. The sequence of problem types in Table 7 can be seen as prototypical of the order in which Donna presented problems for various students; it was developed by examining the actual sequences of problems Donna generated. The numbers that label the various types of addition problems in the table are used in Figure 2 to show the sequences of problems Donna generated for the live and simulated students she tutored. The sequence of problems generated for each student is shown from left to right, with each small box representing a single problem. An asterisk indicates that the simulated student made an error on the problem. The use of praise and instructional moves is also indicated for the simulated tutoring. A move *back* in the sequence of problem types is indicated by a break in the line of boxes. Thus for Student 2 on Day 1, Donna began by giving the student a single problem of Type 3 (two two-digit addends with no carrying). She then gave a Type 4 problem (two two-digit addends with carrying from the

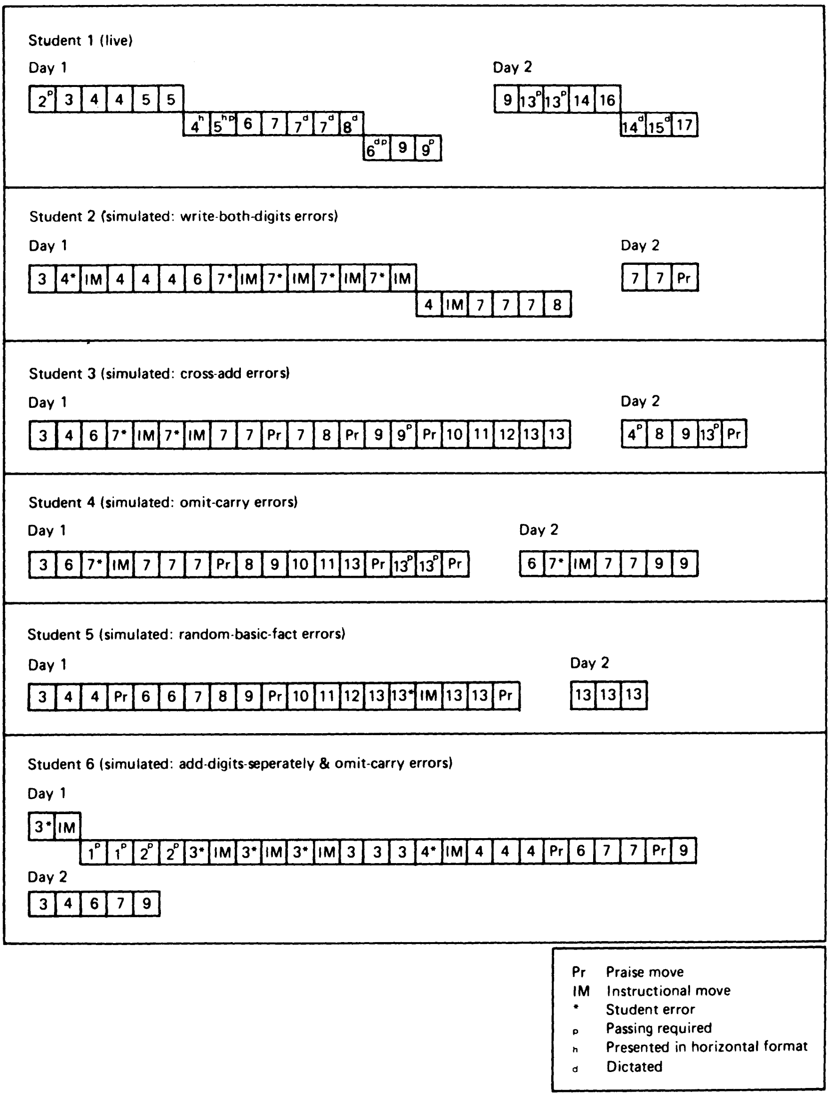

FIGURE 2. *Sequences of problem types presented by Donna during live and simulated tutoring*

ones column) on which Student 2 made a write-both-digits error. Donna responded by selecting an instructional move, followed by three more problems of Type 4. Donna continued to generate problems, responding to student errors with instructional moves. Note that with all of the students, Donna consistently moved forward through the sequence of problem types. She moved back to an earlier type only after student errors

TABLE 7

Addition problem types used by Donna in live and simulated tutoring

Problem type	Number of digits per addend	Number of addends	Carrying from
1	1	2	—
2	1	3–4	—
3	2	2	—
4	2	2	ones
5	2	3	ones
6	3	2	—
7	3	2	ones
8	3	2	tens
9	3	2	ones & tens
10	3	3	—
11	3	3	ones
12	3	3	tens
13	3	3	ones & tens
14	4	2	—
15	4	2	tens
16	4	2	ones & tens
17	4	3	ones & tens

(Students 2 and 6) or to present a problem of an earlier type in a different format (horizontally or dictated for Student 1, the live student).

It was the consistency of the sequences of problems presented by the teachers for different live and simulated students that gave rise to the notion of the *curriculum script* in the model posed as an alternative to the diagnostic/remedial model. Rather than a thorough diagnosis of the students' misconceptions or faulty procedures providing the structure for the topics to be covered, a sequence of problem types—the curriculum script—appeared to be the primary determinant of the agenda for the tutoring sessions. Each teacher moved through this set of problem types, teaching the correct procedures at each step if the student made errors. Although it is impossible to tell whether the problems a teacher presented to a student were predetermined or generated on the spot, the similarity of the orders in which problem features were varied for different students suggests that at least the progression of problem types to be covered was predetermined.

Asking questions. In addition to having the student carry out the steps of the written algorithm, the teachers in the live tutoring used a variety of questions to get students to describe what they were doing or to talk about some concept or process. As with giving students problems to work, the teachers' goals in asking such questions were a combination of seeing

whether students knew or understood procedures or concepts (i.e., determining the state of a student's knowledge) and instructing by providing practice and getting correct information stated. These teacher questions were classified into four types. *Prompting questions* requested the student to carry out some task, usually a part of the addition algorithm. They often occurred as the teacher led the student step by step through a problem—"talking through" the problem—and included oral basic fact questions and requests to read an addend or answer. *Process* or *algorithm questions* focused on having the student talk about the actions that were taken or should be taken in working problems. A teacher's subgoals in asking one of these questions may have included reminding the student about a step to be carried out or seeing if the student could state the steps or rules involved. *Concept* or *understanding questions* dealt either with concepts related to addition or with understanding the addition algorithm in some way. Questions about place value or about concrete objects were included in this category. Teachers used *metacognitive questions* to check on students' confidence to go on or their ability to carry out some procedure ["Do you know how to do that?" (Donna)], requested students to assess their own performance ["Do *you* think you know how to add?" (Carol)], or had students reflect on their own performance ["Is it easier for you to add these two first?" (Fran)].

Table 8 presents the teachers' use of the various types of questions during the live tutoring. Interesting differences among the teachers in their use of process or algorithm questions versus concept or understanding questions can be seen. Donna showed the most extreme preference for process or algorithm questions. Ella similarly preferred process or algorithm questions; many of her student process questions concerned the manner in which her student was summing columns of digits. Ella asked no concept or understanding questions even though her student at times had considerable difficulty with carrying and place-value concepts. Alma and Fran were the only teachers who used more concept or understanding questions than process or algorithm questions. Betty's high proportion of concept or

TABLE 8
Proportions of types of questions during live tutoring

Teacher	Prompting	Process or algorithm	Concept or understanding	Metacognitive
Alma	.39	.11	.26	.24
Betty	.23	.32	.29	.14
Carol	.20	.23	.14	.41
Donna	.00	.85	.06	.09
Ella	.31	.50	.00	.13
Fran	.47	.09	.21	.18

understanding questions was somewhat inflated by a large number of place-value questions that were repeated in problem after problem, making them rather routine and not clearly focused on student understanding of place-value concepts.

Teachers' Overall Goals

Differences among the teachers in their use of questions during the live tutoring reflected their overall goals for teaching addition. Although all of the teachers were given the objective of teaching a student to add two numbers, there was considerable variation in the actual goals teachers had for the tutoring sessions. "Teaching addition" meant different things to different teachers.

The goals of the teachers are summarized in Table 9. All of the teachers had the goal of the student's being able to carry out the steps in the standard written algorithm for addition. They varied considerably, however, in what aspects of the addition procedure they emphasized. Some teachers had algorithmic goals, focusing on carrying out the steps of the algorithm and

TABLE 9
Teachers' overall goals for live tutoring

Teacher	Procedures of written algorithm	Writing problems	Summing digits	Concepts	Other
Donna	Carry out algorithm Know & state rules	Rewrite horizontal problems to vertical format before adding			Nonstandard algorithm for adding in horizontal format
Carol	Carry out algorithm Carrying represents splitting groups of one	Rewrite horizontal problems to vertical format before adding		Zero as place holder	Student should be aware of what she knows and is learning
Ella	Carry out algorithm		Uses strategies for combining digits		
Betty	Carry out algorithm	Can copy problems, keeping digits in proper columns, especially with nonrectangular problems		Place value of digits	Can add numbers mentally
Fran	Carry out algorithm Has manipulative based model of procedures		Knows "secret of" 9, 8, & 7 Uses strategies for combining digits		
Alma	Carry out algorithm Understanding: carrying out algorithms same as combining sets	Can write nonrectangular problems correctly aligned with understanding of place value constraints		Commutativity: reversing addends results in same answer	

knowing the rules needed to carry out these steps. Other teachers had more conceptual goals, emphasizing the student's understanding of procedures or the linking of procedures to manipulatives. The teachers are ordered on the algorithmic-conceptual dimension in Table 9, with Donna falling at the algorithmic end and Fran and Alma at the conceptual end. Differences among the other teachers were not as distinct.

The teachers also addressed different concepts and procedures not directly related to carrying out the addition algorithm. Two teachers, Ella and Fran, focused on students' strategies for combining individual digits when summing a column. Others focused on place value to differing degrees or brought in concepts such as commutativity or zero as a place holder. Two teachers had students use alternative addition procedures, Betty having her student add numbers mentally and Donna teaching a procedure for adding two numbers in horizontal format (e.g., 345 + 54 =).

In the simulated tutoring, teachers could deal only with the actual carrying out of the addition algorithm, precluding attention to the related rules, strategies, or concepts that were addressed in the live tutoring. In spite of these constraints, however, differences in the teachers' use of conceptual and algorithmic moves in the simulated tutoring (presented in Table 10) reflected the overall goals inferred from their live tutoring. With the exception of Ella, the percentages reflected the overall goals of the teachers in the live tutoring. Alma and Fran, whose overall goals emphasized students' concepts and understanding, used more conceptual than algorithmic moves. The other, less conceptually oriented teachers used more algorithmic moves. Ella's exclusive use of *explain* and *concrete* moves in the simulated tutoring was in sharp contrast to her focus in the live tutoring on the student's strategies for combining digits and her avoidance of discussing concepts.

Describing Simulated Students' Faulty Procedures

Additional evidence that the teachers' primary goals in the simulated tutoring were not accurately diagnosing the specific nature of students'

TABLE 10
Percentages of algorithmic and conceptual moves used in simulated tutoring

Teacher	Percentage of algorithmic moves	Percentage of conceptual moves	Total number of instructional moves
Alma	31	69	39
Betty	66	34	44
Carol	94	6	34
Donna	63	38	16
Ella	0	100	10
Fran	18	82	11

difficulties was offered by the teachers' performance in the retroactive prediction task in which they described the difficulties of the simulated students. The teachers were asked to describe each student's difficulty in two ways: (a) verbally describing the difficulties and (b) working various addition problems as the student would have worked them. If the more accurate of the two forms of description is taken in each case, the teachers gave accurate descriptions for 40%, partially accurate descriptions for 23%, and incorrect or no descriptions for 37% of the simulated students. Many of the partially accurate descriptions were due to teachers portraying students' difficulties as being more limited than they actually were (resulting in part from a lack of probing with additional problems to determine the extent of a student's difficulties). For example, Donna said that one simulated student could not carry "from tens to hundreds," when the student actually could not carry in any column.

Teachers often had a hard time remembering the students' difficulties at all. Once a student's faulty procedure was corrected, the teacher moved on to new types of problems and forgot what the student's original difficulty was. The teachers' inaccuracy and difficulty remembering the specific nature of students' faulty procedures suggests that their goals in the simulated tutoring did not center on making accurate and complete diagnoses. This evidence suggests further that the teachers' models of students' knowledge were not detailed models of the students' misconceptions or faulty procedures.

Curriculum Script Model

Because the diagnostic/remedial perspective did not provide an adequate characterization of the tutoring by experienced teachers in this study, the curriculum script concept was developed. A curriculum script is an ordered set of goals and actions for teaching a particular topic—the skills and concepts students are expected to learn and activities and strategies for teaching this material. When teaching a topic, the teacher moves through this curriculum script, gathering information from student performance cues to make minor adjustments in instruction. Diagnosis per se is not a primary goal; it is the teacher's curriculum script, rather than a model of the student based on detailed diagnosis, that determines the sequence of instruction.

Figure 3 presents a model of the curriculum script and other influences on a teacher's agenda for a tutoring session. The model is a refinement of the influences on the teacher's agenda depicted in the general model of tutoring described earlier (see Figure 1). In the refined model, the teacher's curriculum script determines the overall structure of the agenda (the dynamic plan or set of goals and actions for a particular lesson). Student performance cues and the teacher's model of the student are responsible for minor adjustments to the agenda—its microstructure.

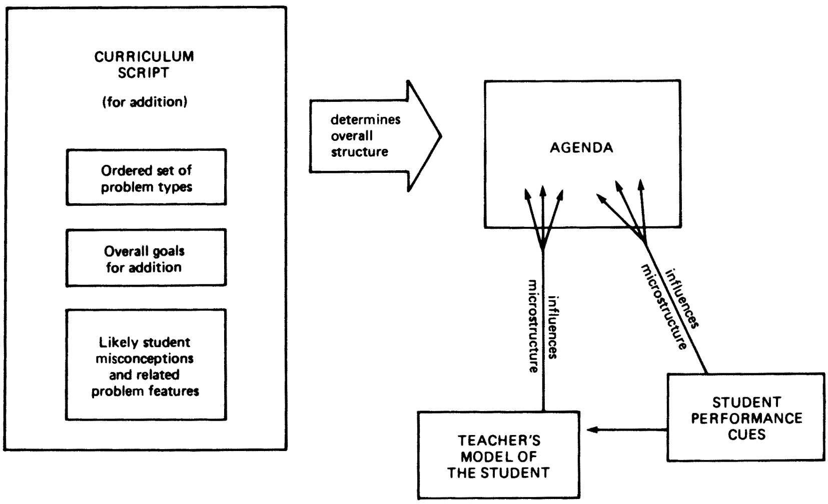

FIGURE 3. *Curriculum script model*

The curriculum script provides the overall structure for the agenda for a particular tutoring session (or lesson). Whereas the agenda, however, is dynamic, changing during the course of a lesson as goals are added and removed, the curriculum script is essentially static. It resides in long-term memory, where it constitutes an important part of the teacher's pedagogical knowledge about addition. The curriculum script can be thought of as the foundation upon which the agenda for a particular lesson is built.

The exact nature of curriculum scripts varies for different subject-matter areas and for different topics, although they always contain loosely ordered sets of goals for what students are to learn, activities and examples to be used in teaching for those goals, and likely student misconceptions or difficulties in learning a particular domain. These are elements of teacher knowledge that fall within *pedagogical subject-matter knowledge* in the general model of tutoring in Figure 1. The curriculum scripts for addition held by the teachers in this study consisted of loosely ordered sets of problem types that students were expected to work, along with overall goals for teaching addition and likely student misconceptions.

Although the curriculum script determines the overall structure of the agenda for a tutoring session, input in the form of student performance cues plays an important role at the more detailed levels of the agenda—its microstructure (see Figure 3). The typical pattern occurring as a teacher moves from one topic or problem type in the curriculum script to the next is illustrated in Figure 4. The teacher has the student work problems of a given type until convinced that performance in this domain is satisfactory. If an error or other difficulty arises, the teacher attempts to correct the

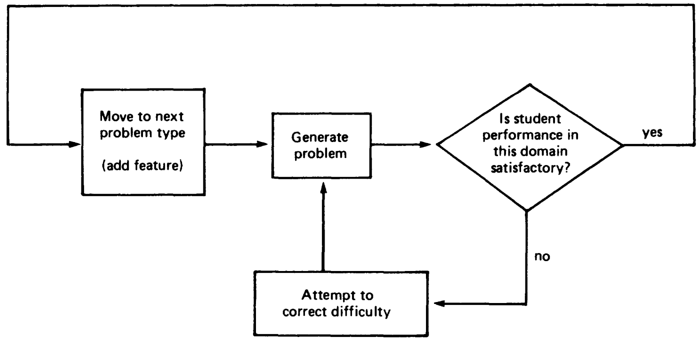

FIGURE 4. *Strategy for moving through a problem type*

difficulty before moving on to the next problem type. Thus the teacher moves through the curriculum script, checking or reviewing the student's knowledge at each stage, and correcting errors as they appear.

This general strategy differs from that suggested by the diagnostic/remedial approach, in which the teacher forms an accurate description of the source of a student's error before attempting remedial instruction. In this case, when student performance is not satisfactory (i.e., an error was made) the teacher would attempt to determine the specific nature of the error, perhaps by generating hypotheses about what the source of the error might be and testing these hypotheses by giving the student more problems to work or by asking questions.

The final influence on the teacher's agenda portrayed in Figure 3 is the teacher's model of the student. It is assumed that the teacher stores and uses information obtained from student performance cues, forming some representation of the knowledge of the individual student during the course of a tutoring session. This model of the student's knowledge influences the microstructure of the agenda by helping to determine whether it is time to move on to new topics, to ask various questions, or to treat a topic in more depth. Although the nature of this model of the individual's knowledge remains uncertain, the current study suggests that it is *not* a highly detailed representation of the student's correct and incorrect conceptions and procedural knowledge.

To summarize, I have presented an alternative to the diagnostic/remedial perspective. In this alternative model, a curriculum script, which is an important part of the teacher's pedagogical knowledge about a topic, determines the overall structure of the teacher's agenda for a particular lesson. The teacher moves through this curriculum script, presenting information, checking the student's knowledge at each stage, and correcting errors as they occur. Thus, minor adjustments are made to the curriculum script on the basis of student performance cues (e.g., errors) and the teacher's model of what the individual student knows.

Discussion

The central conclusion of this study was that the experienced teachers did not attempt to form highly detailed models of their students' knowledge before attempting remedial instruction. Rather, each teacher appeared to move through a curriculum script to teach the individual students. An important issue raised by this use of curriculum scripts concerns why teachers have curriculum scripts. In the live and simulated tutoring the teachers were working with individual students; why did they not spend more time determining the specific strengths and weaknesses of their students before attempting remedial instruction?

One answer lies in the fact that these tutors were experienced classroom teachers. The use of curriculum scripts with individual students was probably an extension of the approach these teachers used in their classroom instruction. With a class of 20 to 30 students, it may well be impossible for a teacher to acquire and use detailed information about skills specific to each student; the cognitive load may simply be too great. Instead, the information needed for teaching a topic may consist largely of subject-matter knowledge organized as a curriculum script. This sequentially organized set of goals and activities provides the basic protocol or agenda for teaching a particular topic. The curriculum script is not student-specific; information about the skills and knowledge of individual students is limited to that required for moving through the script. If curriculum scripts were a significant part of the subject-matter specific pedagogical knowledge used in classroom instruction by the teachers in this study, it seems likely that these curriculum scripts would exert a considerable influence on tutoring in a subject-matter domain that they have taught often. It appears that, even in tutoring settings where higher levels of diagnosis were feasible, the teachers relied on their knowledge of addition content developed by and for classroom teaching.

There are other arguments for the use of curriculum scripts by classroom teachers besides the cognitive load of keeping track in detail of the specific skills of many students. The goal of the classroom teacher regarding a particular subject-matter domain is for students to learn the skills and concepts of that domain. Although the teacher may determine enough about the nature of a student's knowledge to facilitate teaching the subject matter, diagnosis for its own sake is generally not a central goal. It may well be that the classroom teacher can be more successful by focusing cognitive resources on the structuring of the content involved, gearing instruction to the "typical" student, than by devoting major effort to diagnosing the specific knowledge of individuals.

An analogy from the field of medicine may clarify this point. When confronted with an infection, an internist working in an academic setting wants to make a specific diagnosis, determining exactly what bacterium is the source of the illness. A practicing physician treating a patient with the

same infection may want to know only whether the cause is bacterial or viral. If the infection is bacterial, the physician may apply a broad spectrum antibiotic; knowledge of the specific bacterium is not required. Teaching may be similar. If a teacher has an effective sequence of activities for teaching addition, he or she may not need to know the specific nature of difficulties students might be having.

Although useful, this medical analogy should not be carried too far. An important difference between teaching and medicine is that a physician generally tries to correct some disorder to restore a patient to a previous state of health. A teacher, on the other hand, tries to facilitate fundamental changes in the knowledge and thinking of the student—to build up new structures in the mind. It seems appropriate for the teacher to focus on what structures are being built rather than devoting too much effort to the previous states of the learner.

Regarding the issue of what kinds of models of student knowledge teachers have during instruction, this study provided evidence that teachers *do not* form the detailed representations of correct and faulty knowledge of individual students that Brown and Burton (1978) suggested are "a prerequisite to successful remediation" (p. 156). The nature of the models of student knowledge that teachers *do* have, however, remains unclear. There is some evidence suggesting that a teacher locates each student at some point in the curriculum script; the student is likely to know elements of the script that precede that point but not those elements that follow it. In whole number addition, for example, a student who correctly works a problem involving carrying in the tens column is assumed to be able to add problems involving carrying in the ones column. One item of information about the student—his or her location in the curriculum script—thus provides much of what the teacher needs to know about that student's knowledge. The need to organize large amounts of detailed information about the student's knowledge of specific addition subskills is minimized.

Collins and his colleagues (Collins & Stevens, 1982; Collins, Warnock, & Passafiume, 1975) similarly argued that teachers have a partially ordered theory of a subject-matter domain along which they place students. Because all students are thought to learn elements of the domain in approximately the same order, the teacher need only determine a criterion point for each student, separating the elements that are probably known from those that are probably unknown.

Further evidence is offered in a study by Leinhardt (1983), in which teachers were given a copy of a standardized achievement test and asked to report whether individual children had had sufficient instruction to answer each item correctly. In making judgments in this *overlap* task, teachers appeared to locate in the curriculum the skills needed for an item, then locate the child in relation to that point in the curriculum. Sometimes the search for the location of the child was made in relation to other

children in the class. Thus, it seems likely that teachers' information about an individual student's knowledge is structured by their curriculum scripts, although the precise nature of teachers' models of student knowledge should be clarified through further research.

What does this study have to say about teaching practice? If one believes that teachers *should* be highly diagnostic, the finding that the teachers did not focus on determining the exact nature of students' difficulties suggests the need for improving teachers' diagnostic skills. From this point of view, the goal becomes to change teachers—to make them more aware of the misconceptions students might have and to teach them strategies for effective diagnosis.

As argued earlier, however, it may be impossible for a classroom teacher to keep track of each student's skills in detail because of the cognitive load involved. One possible solution for maintaining an instructional approach that addresses specific faulty skills and misconceptions of students within a classroom context is the development of diagnostic aids for the teacher. For example, a computer could be programmed to pinpoint a student's faulty addition skills by posing various problems for the student to solve, resulting in a diagnosis of skill deficiencies and misconceptions given to the teacher. (Similar programs have been developed by Burton, 1981, and Sleeman, 1982.) The teacher presumably could provide instruction pinpointed to specific misconceptions without using large amounts of time for diagnosis.

Although such an approach has intuitive appeal, it might be difficult or impossible to implement successfully. If a teacher's instruction is structured around a curriculum script that does not have a role for detailed diagnosis, it is unlikely that he or she would make use of the specific information provided by a diagnostic aid. A complete restructuring of the teacher's instructional approach would be required.

The role of curriculum scripts suggests that, rather than bettering diagnostic skills, improving the curriculum scripts of teachers would be a way to improve subject-matter instruction. Providing a teacher with increased pedagogical subject-matter knowledge through better curriculum scripts might improve the quality of instruction. Taking this approach would require the specification of effective curriculum scripts for particular topics. What criteria distinguish among more and less effective curriculum scripts? Of particular interest are the curriculum scripts of expert teachers. How do successful teachers in a particular subject-matter area structure the content they teach? To what extent do successful teachers incorporate into their curriculum scripts the variety of misconceptions and faulty procedures children have been shown to have (e.g., Brown & Burton, 1978; Cox, 1974) so that these difficulties are circumvented? Do effective teachers (as determined by student achievement gains or some other criteria) have similar scripts, or are a variety of approaches successful? In teaching

addition, for example, teachers taking a procedure-oriented approach like that of Donna or an understanding-oriented approach like that of Alma might be equally successful in promoting achievement gains in their students. Examining their instruction in terms of curriculum scripts could reveal similarities and differences in the way they structure the content. Examining in detail (perhaps through interviewing) what their students learned could reveal subtle differences in the effect of the different emphases on aspects of student knowledge not revealed by traditional achievement tests.

Comparing in terms of curriculum scripts the instruction of expert and novice teachers might highlight important aspects of effective instruction. How is the expert teacher's organization of subject-matter content different from that of the novice? Do new teachers lack knowledge of activities and examples for communicating a particular topic or are they missing a coherent organization of the subject-matter to be taught? Examining expert and novice teachers should help illuminate the knowledge and organization of that knowledge in the form of curriculum scripts that teachers need to be successful.

Finally, there are two general points to be made about this study. The first concerns the appropriateness of applying a diagnostic perspective like that of clinical medicine to teaching. Such a perspective emerged in studies of teacher thinking that attempted to move beyond the traditional process-product approach by looking at teaching as the gathering of cues and the making of decisions on the basis of those cues (e.g., Shavelson, 1973; Shulman & Elstein, 1975). As discussed earlier, mathematics educators have also argued for diagnosis in teaching (e.g., Ashlock, 1982; Buswell, 1926). In making the case for the existence of curriculum scripts, I presented the diagnostic/remedial approach in an extreme form, with the inclusion of highly detailed models of a student's knowledge and separate phases of diagnosis and remedial teaching. Not all advocates of a diagnostic approach to teaching would insist on these features. What is pervasive in diagnostic approaches, however, is an emphasis on the knowledge and thought processes of the individual student, and the tailoring of instruction to that student. The key feature of the curriculum script model is that it places primary emphasis on the content to be taught—the curriculum—rather than on the idiosyncratic knowledge of individual students. This is not to say that teachers do not adjust instruction for differences in individual students; these adjustments, however, are made within the overall framework of subject-matter content provided by the curriculum script. Both curriculum and adjustment for individuals are needed; I am arguing that we may have placed too much focus on adjustment and diagnosis, and not enough focus on the content to be taught. In the domain of intelligent computer-based instruction, Lesgold (in press) has similarly argued that developers of such systems have neglected the structure of

curriculum by overemphasizing the role of a diagnostic model of the student's knowledge in determining the sequence of instruction.

This study casts doubt on a highly diagnostic perspective for teaching. The image one gains is not of a teacher gathering cues for the purpose of making instructional decisions, but of a teacher moving through a predetermined script of activities, making minor adjustments as needed.

The second point concerns the importance of specific subject-matter content in research on teaching. This study has been one attempt to give detailed attention to the subject-matter content of instruction while focusing on the teacher. Determining the limited role that diagnosis plays for these teachers and developing the concept of curriculum script would have been impossible without detailed examination of the content of the instruction taking place. Continued attention to the subject-matter content of instruction is essential to further elucidate the nature of good teaching. A greater understanding of how teachers organize and structure content to be taught within the constraints of the classroom setting should help us improve *instructional* aspects of teaching by building upon the knowledge base provided by research on teaching in the process-product tradition.

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Author

RALPH T. PUTNAM, Assistant Professor, Michigan State University, Department of Educational Psychology, College of Education, East Lansing, MI 48824. *Specializations*: Research on teaching, mathematics teaching and learning.